INVERSE MULTIGRID CORRECTION FOR GENERALIZED EIGENVALUE COMPUTATIONS

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Summary. The inverse iteration is a classical method for the numerical computation of eigenvalues, which requires the solution of a linear system in each step. For large, sparse systems, a direct solution may be too expensive and iterative solvers must be used. However, the convergence of the outer eigenvalue iteration poses increasing demands on the accuracy of the inner iteration. Thus, high accuracy can be obtained only, when the number of inner iteration steps is increased. We propose a variant of inverse iteration that is based on a correction approach whose accuracy requirements do not increase. This inverse correction method can then be used with only one cycle of a multigrid method for each outer iteration step. If combined with appropriate shifts, this method leads to a rapidly converging overall algorithm. Numerical examples for generalized eigenvalue problems indicate that inverse multigrid correction is an interesting alternative for large sparse eigenvalue problems.

Key words. Inverse Iteration, Eigenvalue Problems, Iterative Methods, Multigrid.

AMS(MOS) subject classifications. 65N55, 65F15, 65N06, 65N25, 82D75

1. Introduction. In this note we study the problem of calculating an eigenvalue-eigenvector pair \((\rho, x)\) for a generalized eigenvalue problem of the form

\[
(L - \rho F)x = 0 \quad \text{with } x \neq 0. \tag{1}
\]

We start our study with with a review of the inverse iteration (cf. [9]) displayed in Algorithm 1. This algorithm requires the inversion of \((L - \tilde{\rho} F)\) in each step of the iteration, where \(\tilde{\rho}\) is an approximation for the eigenvalue \(\rho\). Since we are interested in the case when \(L\) and \(F\) arise from the discretization of partial differential equations (e.g. of the form (11) below) direct solvers are often too expensive. The matrices in our application are most economically inverted by iterative methods. If employed within an outer inverse iteration process, such an iterative solver becomes an inner iteration. Since multigrid methods are asymptotically optimal iterative solvers (see e.g. Hackbusch [10]), they are particularly attractive for the solution of very large systems. Therefore we will use multigrid in the examples in Section 3).

Two general difficulties arise when inverse iteration is combined with iterative inner solvers:

- the better the shift, the closer the system becomes to being singular.
- the accuracy of the inverse iteration is limited by the accuracy of the iterative solution in the inner loop.

Thus we must expect that the inverse iteration requires an increasing number of inner iterations, each of which tends to become more difficult to perform.

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