IMPLICIT EXTRAPOLATION METHODS FOR VARIABLE COEFFICIENT PROBLEMS

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Abstract. Implicit extrapolation methods for the solution of partial differential equations are based on applying the extrapolation principle indirectly. Multigrid tau-extrapolation is a special case of this idea. In the context of multilevel finite element methods, an algorithm of this type can be used to raise the approximation order, even when the meshes are nonuniform or locally refined. The implicit extrapolation multigrid algorithm converges to the solution of a higher order finite element system. This is obtained without explicitly constructing higher order stiffness matrices but by applying extrapolation in a natural form within the algorithm. The algorithm requires only a small change of a basic low order multigrid method.

Key Words. Finite Elements, Extrapolation, Multigrid, Numerical Quadrature.

AMS(MOS) subject classification. 65F10, 65F50, 65N22, 65N50, 65N55.

1. Introduction. Implicit extrapolation is an efficient technique to improve the accuracy of a multilevel solver. When combined with extrapolation, the multilevel principle is not only used as the basis for a fast algebraic solver, but also to increase the approximation order. The basic idea of extrapolation is to exploit discretizations on different levels.

In classical Richardson extrapolation, two or more approximations from different meshes are combined linearly to eliminate the dominating terms of the error expansion. For partial differential equations this has been studied in the context of finite difference discretizations, see e.g. Marchuk and Shaidurov [8] and in the framework of finite elements (FE), see e.g. Blum, Lin, and Rannacher [2]. These techniques are explicit extrapolation methods, since they use approximate solutions directly.

Here we propose a different approach, where extrapolation is applied indirectly to intermediate quantities of the solution process. Such methods are called implicit extrapolation techniques. Methods of this type may be related to defect correction, and — if combined with multigrid — to τ-extrapolation, see e.g. Brandt [3], Hackbusch [5], Schaffer [11], or Bernert [1]. However, these methods are mathematically still motivated by expansions of the truncation error, which in turn require uniform meshes. A generalization to locally uniform meshes can e.g. be found in McCormick and Rüde [9].

In Jung and Rüde [6] we have presented an implicit finite element extrapolation technique which is based on extrapolating the quadrature rules used to compute the stiffness matrices. In [6] it has been shown that within the nested

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